Critical and Flash Points for Metastable Systems in a Mean-Field Approach [*]

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Abstract

By using, as a toy model, an analytical Equation of State which describes a system that can exist in a liquid or vapor phase, we found scaling properties regarding the critical and the "flash" temperature. The "flash" temperature is defined as the highest temperature at which a self-bound system can exist in hydrostatic equilibrium. We conjecture that the "flash" temperature can provide an alternate natural dimension-dependent scale of temperature, other than the critical temperature.

KEY WORDS: equation of state; model; statistical mechanics; vapour-liquid equilibria.

We consider in this letter a simple Equation of State (EOS) derived from a Skyrme-type interaction [1]. Such an interaction is short-ranged and typically used in nuclear physics. Without any loss of generality in our study, it is particularly well suited for our purposes since it allows an analytical derivation of the scaling properties:

$$p = -a_o \rho^2 + 2a_3 \rho^3 + \rho k_B T. \tag{1}$$

This equation shows explicitly a cubic dependence on the density ρ, in much the same way as in the van der Waals fluid. This form for the EOS is typical of a system that can exist in a liquid or vapor phase, and suggests the existence, at low density, of a line of first-order liquid-vapor phase transition in a p versus T phase diagram, ending up at a critical point, where the transition is continuous. The temperature T_c associated with this critical point is an upper bound for the range of temperatures in which the two phases coexist. Many other models for nuclear matter have been studied [2], all of them exhibiting EOS with the same van der Waals fluid-like behavior.

We are aware that the use of a mean-field approach (MFA) has as a drawback that the actual spatial dimension in which the system is embedded loses in part its specificity, as far as the critical behavior is concerned. The critical exponents have the same values in all dimensions, and the EOS collapse into one single curve in the neighborhood of criticality, when rescaled with the critical parameters. For a simple van der Waals gas this collapse occurs throughout all the range of the thermodynamic variables, as can be trivially verified [3]. The same is true for the analytically soluble model with Skyrme-type interactions above mentioned.

Mean-field phase diagrams can be valuable, even below the upper critical dimension of a model, to explore its regions of meta-stability. As is well known from the study of fluids, a physical system can get trapped in a local minimum of its free energy, from which it escapes only after a finite time. In an MFA the position of these local minima are bounded in the phase diagram by the spinodal curves, which lie inside the region of phase coexistence. One usually considers the isothermic, for quenches through processes at constant temperature,

and adiabatic spinodals, which can be formally determined by the solutions of $\partial p/\partial \rho = 0$, keeping constant the appropriate thermodynamic variable. The processes of fragmentation and super-heating are associated with the regions $\partial p/\partial \rho < 0$ and $\partial p/\partial \rho > 0$ respectively.

A particularly interesting region still inside the coexistence region is that in which a hydrostatic equilibrium $(p = 0)$ is still possible and the nuclear matter incompressibility

$$K(T) = \left.\frac{\partial p}{\partial \rho}\right|_{p=0} \tag{2}$$

can be calculated. This region is delimited by $0 < T < T_{fl}$ and $\rho_{fl} < \rho < \rho_o$, where the point (ρ_{fl}, T_{fl}) is obtained as the solution to $p = \partial p/\partial \rho = 0$ and is known as the "flash" point. This point represents the smallest density and the highest temperature at which a self-bound system can exist in hydrostatic equilibrium, and belongs by definition to a spinodal. It can thus provide an alternate natural dimension-dependent scale of temperatures, other than the critical temperature. It is worth mentioning at this point that critical behavior at spinodal points has been found in the study of mean-field versions of classical models in condensed matter, such as the inconspicuous Ising model [4].

Here we proceed the discussion regarding the scaling with critical and flash parameters through the simple analytical EOS , given by Eq. 1 [1],

Although derived for a $3D$ system [5], a straightforward reproduction of that reasoning can be used to show that it has the same functional form in every spatial dimension. The relation between its coefficients and those of the interaction potential are dimension-dependent though. These coefficients have dimensions $[a_0] = M^{D+1}$ and $[a_3] = M^{1-2D}$. The density at saturation is obtained as the solution to $p(\rho_0) = 0$, leading to $\rho_0 = \frac{a_0}{2a_3}$. The incompressibility (at saturation) is obtained as

$$K(T) = \left.\frac{\partial p}{\partial \rho}\right|_{p=0}, \tag{3}$$

which yields

$$K(T) = \frac{a_0^2}{4a_3}\left[\,1 + \sqrt{1 - \frac{8a_3 k_B T}{a_0^2}} - \frac{8a_3 k_B T}{a_0^2}\,\right] \tag{4}$$

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and $K(0) = \frac{a_0^2}{2a_3}$. The reduced incompressibility can thus be written as

$$\frac{K(T)}{K(0)} = \frac{1}{2} \left[1 + \sqrt{1 - \frac{8a_3 k_B T}{a_0^2}} - \frac{8a_3 k_B T}{a_0^2} \right].$$

(5)

We begin by deriving a law of corresponding states for this *EOS* by rescaling the thermodynamical variables with their critical values. The critical point in which the liquid-vapor coexistence phase disappear and matter starts to be described as a gas is obtained via

$$\left.\frac{\partial p}{\partial \rho}\right|_{\rho=\rho_c} = \left.\frac{\partial^2 p}{\partial \rho^2}\right|_{\rho=\rho_c} = 0$$

(6)

leading to

$$\rho_c = \frac{a_o}{6a_3} \quad , \quad k_B T_c = \frac{a_o^2}{6a_3} \quad , \quad p_c = \frac{a_o^3}{108 a_3^2}$$

(7)

with

$$a_o = \frac{k_B T_c}{\rho_c} \quad , \quad a_3 = \frac{k_B T_c}{6\rho_c^2}.$$

(8)

Substituting the values of a_o and a_3 into Eqs. 1 and 5 one obtains

$$p' = \rho'^3 - 3\rho'^2 + 3\rho'T'$$

(9)

and

$$\frac{K(T)}{K(0)} = \frac{1}{2} \left[1 + \sqrt{1 - \frac{4}{3}T'} - \frac{4}{3}T' \right].$$

(10)

where $p' = p/p_c$, $\rho' = \rho/\rho_c$ and $T' = T/T_c$.

In this particular case, $p_c/k_B T_c \rho_c = 1/3$, near the $3/8$ value obtained for the van der Waals gas. Eq. 9 is an expression of a law of corresponding states valid across different spatial dimensions.

Now, let us show that a similar law can be obtained when the variables are rescaled through their "flash point" values. At this point,

$$\left.\frac{\partial p}{\partial \rho}\right|_{\rho=\rho_f} = p(\rho_f) = 0.$$

(11)

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Imposing the above conditions on Eq. 1 we find

$$\rho_f = \frac{a_o}{4a_3} \quad , \quad k_B T_f = \frac{a_o^2}{8a_3} \quad , \quad p_f = 0 \tag{12}$$

with

$$a_o = \frac{2k_B T_f}{\rho_f} \quad , \quad a_3 = \frac{k_B T_f}{2\rho_f^2}, \tag{13}$$

which when substituted back into Eqs. 1 and 5 leads to

$$p^* = \rho^{*3} - 2\rho^{*2} + \rho^* T^*, \tag{14}$$

and

$$\frac{K(T)}{K(0)} = \frac{1}{2} \left[1 + \sqrt{1 - T^*} - T^* \right]. \tag{15}$$

where

$$p^* = p/k_B \rho_f T_f \quad , \quad \rho^* = \rho/\rho_f \quad , \quad T^* = T/T_f . \tag{16}$$

Here, p^* does not scale with the "flash" parameter p_f which is identically zero by construction, but with $k_B \rho_f T_f$ instead.

We can see in this case that $\frac{T_c}{T_f} = \frac{4}{3}$. Eqs. 10 and 15 relate the incompressibility curves in terms of different criticalities. Therefore, we found that the flash temperature can provide an alternate natural dimension-dependent scale of temperature, other than the critical temperature.

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